Transverse Poisson structures to adjoint orbits in semi-simple Lie algebras

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Abstract

We study the transverse Poisson structure to adjoint orbits in a complex semi-simple Lie algebra. The problem is first reduced to the case of nilpotent orbits. We prove then that in suitably chosen quasi-homogeneous coordinates the quasi-degree of the transverse Poisson structure is -2. In the particular case of $s$ubregular nilpotent orbits we show that the structure may be computed by means of a simple determinantal formula, involving the restriction of the Chevalley invariants on the slice. In addition, using results of Brieskorn and Slodowy, the Poisson structure is reduced to a three dimensional Poisson bracket, intimately related to the simple rational singularity that corresponds to the subregular orbit.